

$$1. \lim_{x \rightarrow 0} \frac{\sin(2x)}{3x}$$

Soln.

We use L'Hopital's rule

$$\lim_{x \rightarrow 0} \frac{\frac{d}{dx}(\sin(2x))}{\frac{d}{dx}(3x)}$$

We get the derivatives ↓

$$\Rightarrow \lim_{x \rightarrow 0} \frac{2 \cos(2x)}{3}$$

We evaluate the limit as $x \rightarrow 0$

$$\Rightarrow \frac{2 \cos(2 \times 0)}{3} \Rightarrow \cos(0) = 1$$

$$\Rightarrow \frac{2}{3}$$

$$2. \lim_{x \rightarrow \infty} \frac{6x^4 + x^2 + 2}{5x^4 - 2x^3 + 4x}$$

Soln.

We evaluate the numerator and denominator separately.

Numerator.

$$\lim_{x \rightarrow \infty} 6x^4 + x^2 + 2$$

We evaluate the limit.

Denominator.

$$\lim_{x \rightarrow \infty} 5x^4 - 2x^3 + 4x$$

We transform the expression by factoring.

$$\lim_{x \rightarrow \infty} \frac{6x^4 + x^2 + 2}{5x^4 - 2x^3 + 4x}$$

on factoring the expression we have ↓

$$\lim_{x \rightarrow \infty} \frac{x^4 \left(6 + \frac{1}{x^2} + \frac{2}{x^4} \right)}{x^4 \left(5 - \frac{2}{x} + \frac{4}{x^3} \right)}$$

Since x^4 is common in both numerator & denominator we cancel them out. To have ↓

$$\lim_{x \rightarrow \infty} \frac{6 + \frac{1}{x^2} + \frac{2}{x^4}}{5 - \frac{2}{x} + \frac{4}{x^3}}$$

⇒ Evaluate the limit. We have.

$$\frac{6 + 0 + 2 \times 0}{5 - 2 \times 0 + 4 \times 0} = \frac{6}{5} = \frac{1}{5}$$

$$3. \lim_{x \rightarrow 6} \frac{3-x}{6-x}$$

$$\text{Soln.} \Rightarrow (3-x) \left(\frac{1}{6-x} \right)$$

$$\therefore \lim_{x \rightarrow 6} (3-x) \left(\frac{1}{6-x} \right)$$

$$\lim_{x \rightarrow a} [f(x) \cdot g(x)] = \lim_{x \rightarrow a} f(x) \cdot \lim_{x \rightarrow a} g(x)$$

$$\Rightarrow \lim_{x \rightarrow 6^+} (3-x) \cdot \lim_{x \rightarrow 6^+} \left(\frac{1}{6-x} \right)$$

$$\Rightarrow \lim_{x \rightarrow 6^+} (3-x) = -3$$

$$\Rightarrow \lim_{x \rightarrow 6^+} \left(\frac{1}{6-x} \right) = -\infty$$

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$\Rightarrow$ Apply infinity property.
 $\Rightarrow -\infty \cdot (-\infty) = \infty$

$$\therefore (-3)(-\infty) = \underline{\underline{\infty}}$$

$$x+3=0, \Rightarrow x = \underline{\underline{-3}}$$

$$x = \underline{\underline{-3}}, x \neq \underline{\underline{4}}, x \neq \underline{\underline{-6}}$$

$$4. f(x) = \frac{x^2 + 9x + 18}{x^2 + 2x - 24}$$

Soln

Let $f(x) = 0$.

$$0 = \frac{x^2 + 9x + 18}{x^2 + 2x - 24}$$

The defined range, $x \neq 4, x \neq -6$.

We rewrite $9x$ as a sum.

$$\Rightarrow 0 = \frac{x^2 + 6x + 3x + 18}{x^2 + 2x - 24}$$

$$\Rightarrow 0 = \frac{x^2 + 6x + 3x + 18}{x^2 + 6x - 4x - 24}$$

We factor the expression.

$$0 = \frac{x(x+6) + 3(x+6)}{x(x+6) - 4(x+6)}$$

$\Rightarrow$

$$0 = \frac{\cancel{x+6}(x+3)}{\cancel{x+6}(x-4)}$$

$$\Rightarrow \frac{x+3}{x-4} = 0$$

$$\Rightarrow \lim_{x \rightarrow 6^+} (3-x) = -3$$

$$\lim_{x \rightarrow 6^+} \left(\frac{1}{6-x} \right) = -\infty$$

$\Rightarrow$ Apply infinity property.
 $\Rightarrow -6 \cdot (-\infty) = \infty$

$$\therefore (-3)(-\infty) = \underline{\underline{\infty}}$$

$$x+3=0, \Rightarrow x = \underline{\underline{-3}}$$

$$\underline{\underline{x = -3}}, \underline{\underline{x \neq 4}}, \underline{\underline{x \neq -6}}$$

Vertical asymptote = $x = \underline{\underline{4}}$.

Discontinuity $x \neq 4$ and $x \neq 6$

No critical points.

$$4. f(x) = \frac{x^2 + 9x + 18}{x^2 + 2x - 24}$$

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Let $f(x) = 0$.

$$0 = \frac{x^2 + 9x + 18}{x^2 + 2x - 24}$$

The defined range, $x \neq 4, x \neq -6$.
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We factor the expression.

$$0 = \frac{x(x+6) + 3(x+6)}{x(x+6) - 4(x+6)}$$

$\Rightarrow$

$$0 = \frac{\cancel{x+6}(x+3)}{\cancel{x+6}(x-4)}$$

$$\Rightarrow \frac{x+3}{x-4} = 0$$

$$5. f(x) = \begin{cases} 3x+2, & x < 2 \\ x^3, & x \geq 2 \end{cases}$$

The function is discontinuous at $x = 2$.

Soln.

$$\left. \begin{aligned} f(x) &= 3(2) + 2 = 6 + 2 = 8 \\ f(x) &= 2^3 = 8 \end{aligned} \right\} \text{The limit exists.}$$